Find the exact value of each expression.

1. \( \sin \frac{5\pi}{3} = -\frac{\sqrt{3}}{2} \)
2. \( \sec \frac{3\pi}{4} = -\sqrt{2} \)
3. \( \tan \left( \frac{5\pi}{6} \right) = \sqrt{3} \)
4. \( \cos \left( -\frac{11\pi}{6} \right) = \frac{\sqrt{3}}{2} \)
5. \( 9 \csc \frac{\pi}{3} - 8 \cot \frac{5\pi}{6} = 9 \left( \frac{2\sqrt{3}}{3} \right) - 8 \left( -\sqrt{3} \right) \)

6. A right triangle has an acute angle \( \theta \) such that \( \csc \theta = \frac{7}{3} \). Find \( \tan \theta \).

7. A right triangle has an acute angle \( \theta \) such that \( \sin \theta = \frac{7}{9} \). Find \( \tan \theta \).

8. If \( \cos \theta = \frac{3}{4} \) and \( \tan \theta < 0 \), find the value of the other 5 trigonometric functions.

Solve each triangle.

9. \( \alpha = 32^\circ \)
   \( \beta = 43.40^\circ \)
   \( \gamma = 30.81^\circ \)

10. \( \alpha = 37^\circ \)
    \( \beta = 43\gamma^\circ \)
    \( \gamma = 37 + 43 = 80^\circ \)

11. An object is traveling around a circle with a radius of 20 meters. If in 10 seconds a central angle of \( \frac{1}{3} \) radian is swept out, what is the linear speed of the object?

\[
\omega = \frac{1}{\frac{1}{3}} \text{ rad} = \frac{3}{10} \text{ rad/sec}
\]

\[
\nu = 20 \left( \frac{3}{10} \right) = \frac{3}{2} \text{ meters/sec}
\]

12. A railroad track is laid along the arc of a circle of radius 1800 ft. The circular part of the track intercepts a central angle of 40°. How far does a train travel on this part of the track? If it is traveling 30 mph then how long will it take to complete this part of the track?

\[
\theta = 40^\circ, \quad r = \frac{2\pi}{40} \quad s = (1800)(\frac{2\pi}{40}) = 400\pi \text{ ft}.
\]

\[
0.24 \text{ miles} = 0.0079 \text{ hrs} = \frac{5280}{1} \text{ ft/h}, \quad 1 \text{ mph} = \frac{1}{1} \text{ hr/mile} = \frac{1}{1} \text{ mi/hr} = \frac{5280}{1} \text{ ft/hr}.
\]

13. The wheels with a radius of 15 inches are spinning at 60 rpm. How fast in mph is the vehicle moving?

\[
\omega = 60 \text{ rev/min}, \quad \frac{2\pi}{1 \text{ rev}} = 120\pi \quad v = 15(120\pi) = \frac{1800\pi}{120} \text{ in./min}, \quad 1 \text{ ft} = \frac{1}{12} \text{ in.}, \quad 1 \text{ mile} = \frac{5280}{1} \text{ ft}, \quad \text{mph} = \frac{1}{1} \text{ hr/mile} = \frac{5280}{1} \text{ ft/hr}.
\]

14. The pilot of an airplane flying 12,000 feet sight a water tower. The angle of depression to the base of the tower is 25°. What is the length of the line of sight from the plane to the tower?

\[
\sin 25^\circ = \frac{25}{12000} \quad x = 28394.42 \text{ feet}
\]

15. The angle of elevation from a point 100 feet from the base of a school to the top of a flagpole on the school is 39°. The angle of elevation from the same point to the base of the flagpole is 28°. What is the height of the flagpole?

\[
\tan 28 = \frac{x}{100} \quad x = 53.17 \text{ feet}
\]

\[
\tan 39 = \frac{y}{100} \quad y = 80.98 \text{ feet}
\]

\[
h = y - x
\]

\[
h = 27.81 \text{ feet}
\]
TRIGONOMETRY
REVIEW CHAPTER 2

NAME ____________________________ KEY ____________________________

Convert each angle in degrees to radians.
1. \(225^\circ\) \(\frac{5\pi}{4}\)
2. \(300^\circ\) \(\frac{5\pi}{3}\)
3. \(270^\circ\) \(\frac{3\pi}{2}\)

Convert each angle in radians to degrees.
4. \(\frac{6\pi}{7}\) \(154.3^\circ\)
5. \(\frac{5\pi}{6}\) \(150^\circ\)
6. \(\frac{\pi}{2}\) \(90^\circ\)

7. Give the formulas for a 6 trigonometric functions in terms of \(x, y,\) and \(r.\)

\[
\sin \theta = \frac{y}{r} \quad \cos \theta = \frac{x}{r} \quad \tan \theta = \frac{y}{x} \\
\csc \theta = \frac{r}{y} \quad \sec \theta = \frac{r}{x} \quad \cot \theta = \frac{x}{y}
\]

Find the exact value of each expression. Do NOT use a calculator!

8. \(\sin 120^\circ = \frac{\sqrt{3}}{2}\)
9. \(\tan 210^\circ = \frac{-\sqrt{3}}{3}\)
10. \(\cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}\)
11. \(\cot(-135^\circ) = 1\)

12. \(\csc \frac{\pi}{3} = \frac{2\sqrt{3}}{3}\)
13. \(\tan(-30^\circ) = -\frac{\sqrt{3}}{3}\)
14. \(\sec \frac{5\pi}{6} = \frac{2\sqrt{3}}{3}\)
15. \(\tan \frac{\pi}{2} = \text{undefined}\)

16. \(\cos \frac{\pi}{2} = 0\)
17. \(\sin \pi = 0\)
18. \(\cot 2\pi = \text{undefined}\)
19. \(\sin \frac{3\pi}{2} = -1\)

20. \(\tan 480^\circ = -\sqrt{3}\)
21. \(\csc \frac{13\pi}{6} = 2\)

22. \(\tan \frac{\pi}{3} + \csc \frac{\pi}{3} = \frac{5\sqrt{3}}{3}\)
23. \(3\csc \frac{\pi}{4} - 2\cot \frac{\pi}{6} = \frac{3\sqrt{2} - 2\sqrt{3}}{3}\)

24. Explain why \(\cot 40^\circ = \frac{\sin 50^\circ}{\sin 40^\circ}\) is undefined.

25. Explain why \(\tan 25^\circ \cdot \sec 65^\circ \cdot \cos 25^\circ = 1\)

26. If \(\sin \theta = \frac{12}{13}\) and \(\cos \theta < 0,\) find the exact value of each.
   \[\cos \theta = \frac{5}{13}, \quad \tan \theta = \frac{12}{5}, \quad \csc \theta = \frac{13}{12}\]
   \[\sec \theta = \frac{13}{5}, \quad \cot \theta = \frac{5}{12}\]

27. If \(\tan \theta = -\frac{1}{2}\) and \(\cos \theta < 0,\) find the exact value of each.
   \[\sin \theta = \frac{\sqrt{5}}{5}, \quad \cos \theta = \frac{-2\sqrt{5}}{5}, \quad \csc \theta = \frac{\sqrt{5}}{1}\]
   \[\sec \theta = \frac{2}{\sqrt{5}}, \quad \cot \theta = -\frac{\sqrt{5}}{2}\]
30. A windshield wiper of a car is 14 inches long. If it takes 1 second to make 1/3 revolution, how fast (v) is the tip of the wiper moving? (Leave your answer in inches per second.)

(Hint: You are given ω in revolutions per second.)

\[ \omega = \frac{1 \text{ rev}}{1 \text{ sec}} \times \frac{2\pi}{1 \text{ rev}} = \frac{2\pi}{3} \text{ rad/ sec} \]

\[ v = 14 \left( \frac{2\pi}{3} \right) = \frac{28\pi}{3} \approx 29.32 \text{ in/ sec} \]

31. Find the height of a building if the angle of elevation is 24° from a horizontal distance of 120 ft.

\[ \tan 24° = \frac{h}{120} \]

\[ h = 53.43 \text{ feet tall} \]

32. Measuring the length of a lake. From a stationary hot air balloon 350 feet above the ground, two sightings of a lake are made. How long is the lake?

\[ \tan 32° = \frac{x}{350} \quad x \approx 218.70 \]

\[ \tan 54° = \frac{y}{350} \quad y \approx 481.73 \]

\[ \text{lake} = y - x \]

\[ \text{lake} = 263.03 \text{ feet long} \]

33. Find the arc length and the area of the sector with a radius of 18 feet and a central angle of 130 degrees.

\[ s = r \theta \]

\[ s = 18 \left( \frac{13\pi}{18} \right) \]

\[ s = 13\pi \approx 40.84 \text{ ft} \]

\[ A = \frac{1}{2} r^2 \theta \]

\[ A = \frac{1}{2} (18)^2 \left( \frac{13\pi}{18} \right) \]

\[ A = 117\pi \approx 367.57 \text{ ft}^2 \]